 Sub Code:RAS103

Paper Id: 199106 **Roll No:**

B. TECH
(SEM I) THEORY EXAMINATION 2019-20
ENGINEERING MATHEMATICS-I

Time: 3 Hours *Total Marks: 70*

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt ***all*** **questions in brief.** **2 x 7 = 14**

a.	For what value of 'λ', the vectors $(1,-2,\lambda),(2,-1,5)$ and $(3,-5,7\lambda)$ are linearly dependent.
b.	If A is a skew-Hermitian matrix, then show that iA is Hermitian.
c.	Find the maximum value of the function $f(x,y,z)=(z-2x^2-2y^2)$ where $3xy-z+7=0$.
d.	Evaluate $\int_0^1\int_0^{x^2} e^{y/x}\,dydx$
e.	Show that the vector field $\vec{V}=(\sin y+z)\hat{i}+(x\cos y-z)\hat{j}+(x-y)\hat{k}$ is irrotational.
f.	Find the area bounded by the parabola $y^2=4ax$ and its latus rectum. .
g.	For the scalar field $u=\frac{x^2}{2}+\frac{y^2}{3}$, find the magnitude of gradient at the point(1,3)

SECTION B

2. Attempt any ***three*** **of the following:** **7 x 3 = 21**

a.	(i)Express $A^6-4A^5+8A^4-12A^3+14A^2$ as a linear polynomial in A where A=$\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$. **(3)** (ii) Reduce the matrix A=$\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$. to diagonal form. **(4)**
b.	(i)Trace the curve $r=2a\cos\theta$ (3) (ii)If $u=\operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$ prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}(13+\tan^2 u)$ (4)
c.	(i) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. (3) (ii)If $u=x^3+x^2y+x^2z-z^2(x+y+z)$, $v=x+z$, $w=x^2-z^2+xy-zy$ Prove that u,v and w are connected by a functional relation (4)
d.	(i) Change the order of integration for I = $\int_0^1\int_{x^2}^{2-x} xy\,dxdy$ and hence evaluate the same. (3) (ii) A triangular prism is formed by planes whose equations are $ay=bx, y=0$ and $x=a$. Find the volume of the prism between the planes $z=0$ and surface $z=c+xy$. (4)

 Sub Code:RAS103

Paper Id: 199106 **Roll No:**

e.	(i) If $\vec{F} = 3xy\,\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}.dr$, where C is the arc of the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$. (3) (ii) Find the directional derivative of $\frac{1}{r}$ in the direction of $\vec{r}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (4)

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

(a)	If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and calculate $y_n(0)$.
(b)	If $x = e^r\cos\theta,\ y = e^r\sin\theta$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2}\right)$.

4. Attempt any *one* part of the following: 7 x 1 = 7

(a)	If $f(x,y) = x^2 y^{1/10}$, compute the value of f when $x = 1.99$ and y = 3.01.
(b)	A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

5. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Find the Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
(b)	Find non-singular matrices P and Q such that PAQ is in normal form of the matrix and hence find the rank of matrix A= $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

6. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Find the volume of the portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant.
(b)	Transform the double integral $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dydx}{\sqrt{a^2-x^2-y^2}}$ into polar coordinates and then evaluate it.

7. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Verify Gauss divergence theorem for $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ over the parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$.
(b)	Evaluate the integral $\int_C \{(x+y)dx + (2x-z)\,dy + (y+z)\,dz\}$ where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0)$ and $(0,0,6)$ using stoke's theorem.